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GEOPHYSICS

Hydrodynamic Instability at Mid-Latitudes of the Earth Caused by the Rotation of the Planet

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For almost 100 years the scientific community has been trying to solve the intriguing problem of the "critical latitudes" [11]. According to the theoretical studies in the field of precession, nutation, and the exact form of the Earth, a stressed state of the planet exists in the zone of the 35th latitude (in the Northern and Southern hemispheres). In these zones (±35°) the main tensions lead to the formation of the deep fracture system and folding in the crust layer [3, 4]. However, the problem about the reasons and physical mechanism that cause the fact that the critical latitudes play a special role in the formation of the Earth's topography and manifestation of the seismic activity remains open. The objective of this work is to present a model describing the development of two symmetric zones of hydrodynamic instability with respect to the equator and to show the manifestations of this instability on the basis of the recent geophysical observations. These zones are caused by the rotation of the planet. They are located at mid-latitudes.

V.E. Khain and N.V. Koronovskii [6] wrote that increased tectonic and magmatic activity of the Earth's interior in the band between 30° of the northern and southern latitude is related to the accelerated rotation of this band relative to the polar regions. They also showed that this factor was active already at the early stage of our planet's development. The analysis of the accumulated material in geology and geophysics, as well as the modern concepts about the effects of hydrodynamic instability [5], allowed the authors to put forward the following hypothesis.

Three characteristic regions are distinguished in any heavy rotating ball (planet): two symmetrically located polar regions, each of them occupying the space from the pole to a specific critical mid-latitude and one near-equatorial region confined between the critical latitudes. The difference in the linear velocities of the particle motions in the near-equatorial and polar regions (due to the strong difference in the mean radii of the regions) leads to the appearance of a zone of hydrodynamic instability at the boundary between the two neighboring regions with strongly different velocities of the particle motion. The development of instability in such zones should be manifested in the formation of turbulence generating perturbations in the forming solid shell of the Earth observed as latitudinal systems of fractures and regions of high seismic activity.

A theoretical model was suggested to justify the main statements of the hypothesis, and a problem was considered for calculating the variations in the inertial moment of a rotating planet as a function of the geocentric latitude for the homogeneous Earth and layered inhomogeneous by the density of the Earth.

Any elastic solid material body rotating around a given axis can be presented as an infinite sum of rotating infinitely thin circular disks with thickness dz and radius r comprising the body. Each of the disks is perpendicular to the rotation axis (Fig. 1a). Then, owing to the additivity property, the inertial moment of the body would be equal to the sum of the inertial moment of the rotating disk applied to the case of a rotating piecewise inhomogeneous elastic Earth.

We shall consider that ellipsoid of revolution is an approximation of the geoid.

Let us determine the inertial moment and variation of rotation inertia of this ellipsoid as a function of height calculated from the equatorial plane to the pole. In the first stage, we shall consider a model of the Earth with homogeneous density (ρ).

The inertia moment $j(z)$ of a circular disk with radius r with constant surface density σ is determined by the following relation:

$$j(z) = \frac{\pi}{2} r^4 \sigma = \frac{\pi}{2} R^4 \left(1 - \frac{z^2}{H^2}\right)^2 \sigma, \qquad (1)$$

where R and H are the equatorial and polar radii of the Earth, respectively. Then $\sigma = \rho dz$; compression of the ellipsoid is $\varepsilon = \frac{R - H}{R}$; and the inertial moment of a

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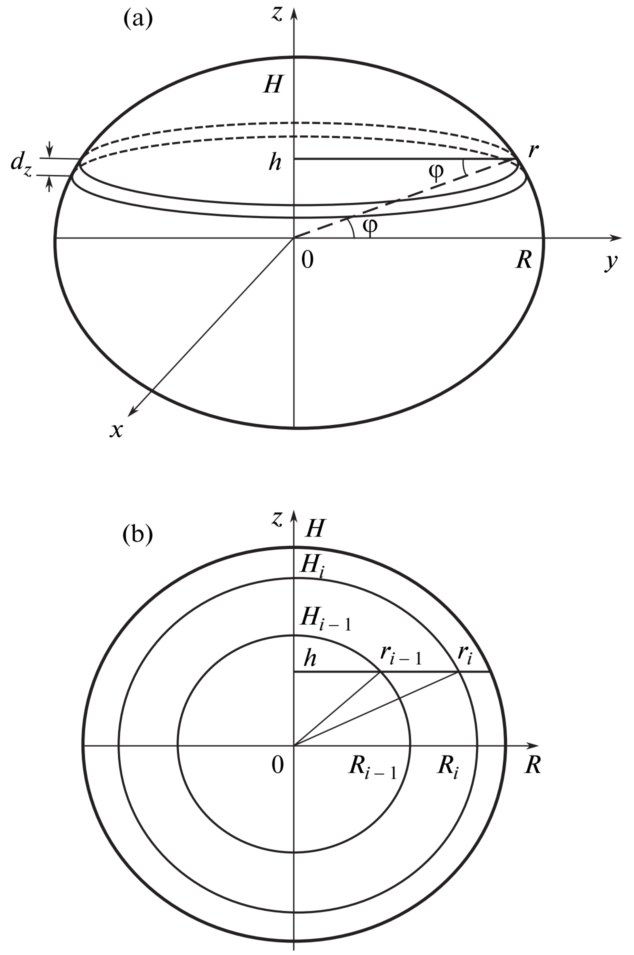

Fig. 1. Illustration of the disk model of homogeneous (a) and layered-heterogeneous (b) Earth.

layer with thickness h, $0 \le h \le H$ of the ellipsoid of revolution can be written as follows assuming that $t = \dfrac{h}{H}$

$$J(h) = \frac{\pi}{2}\rho R^4 \int_0^h \left(1 - \frac{z^2}{H^2}\right)^2 dz =$$

$$= \frac{\pi}{2}\rho R^5 (1-\varepsilon)\left(t - \frac{2}{3}t^3 + \frac{1}{5}t^5\right) = \frac{15}{16} J_0 \left(t - \frac{2}{3}t^3 + \frac{1}{5}t^5\right), \quad (2)$$

$$J_0 = \frac{8\pi}{15}\rho R^4 H = \frac{8\pi}{15}\rho R^5 (1-\varepsilon), \quad (3)$$

where J_0 is the inertial moment of the ellipsoid of revolution with half-axes R and H and density ρ.

We assume that $\zeta(t) = t - \frac{2}{3}t^3 + \frac{1}{5}t^5$ and the second derivative is $\zeta''(t) = -4\left(t - t^3\right) \le 0$; therefore, function $\dfrac{dJ}{dh}$ decreases with increasing h. A graph of function $\dfrac{dJ}{dh}$ will have an inflection point $h_* = \dfrac{H}{\sqrt{3}}$. This point corresponds to latitude $\varphi^* = 35°15'22''$ (Fig. 1a). According to the construction $j(h) = dJ(h)$; then, the inertial moment of a thin circular disk has the same inflection point, in which the value of the inertial moment changes the negative acceleration of decreasing to the corresponding positive acceleration.

Thus, the critical geocentric latitude is the inflection point for the graph of the inertial moment for an infinitely thin layer of the Earth. In this case, the inertial moment is determined as a function of the distance from the equatorial plane to a parallel plane, which crosses the globe at the given latitude.

It is worth noting that this value of the geocentric latitude coincides with the value obtained using another method, which is described in [4].

It was assumed in the calculations that the mean density of the Earth is $\rho = 5.5145 \cdot 10^3$ kg/m^3, the equatorial radius is $R = 6.378 \cdot 10^6$ m, and compression is $\varepsilon = 0.003353$. Then the inertial moment of the homogeneous Earth is $J_0 = 9.720 \cdot 10^{37}$ kg m^2.

Let us consider the case of a layered inhomogeneous Earth presented as n ellipsoid shaped layers of constant density with the same compression ε (Fig. 1b). Each such layer is the difference of the volumes of the two corresponding ellipsoids of revolution. Let us determine the inertial moment of the rotation of such a body as the function of height calculated along the beam from the center of the Earth to the North Pole.

Here, R_i, H_i are the major and minor axes of the ellipsoids; $R_n \equiv R$ is the equatorial, and $H_n \equiv H$ is the polar radii of the Earth. The inertial moment of rotation j_i^0 for an infinitely thin circular disk of density ρ_i with thickness dz and radius r_i is determined in relation (1).

We shall consider that compressions $\varepsilon = \dfrac{R_i - H_i}{R_i}$ in all layers are the same. Then, the inertia moment of an infinitely thin ring with inner and outer radii r_{i-1} and r_i is

$$j_i(z) = j_i^0(z) - j_{i-1}^0(z) = \frac{\pi}{2}\left(r_i^4(z) - r_{i-1}^4(z)\right)\rho_i dz,$$

while the total inertial moment $j_s(z)$ for n of such rings is determined by equation $j_s(z) = \sum_{i=1}^{n} j_i(z)$. After the change of variables $t = \dfrac{h}{H}$, $\xi = \dfrac{z}{H}$, $\beta = \dfrac{H_i}{H}$, $\gamma_i = \dfrac{\rho_i}{\rho}$, and integration, we get

$$J_s(h) = \frac{15}{16} J_0 \left(\gamma_n \zeta(t) + \sum_{i=1}^{n-1} (\gamma_i - \gamma_{i+1}) \lambda(t, \beta_i)\right),$$

$$\lambda(t, \beta_i) = \begin{cases} \beta_i^4 t - \frac{2}{3}\beta_i^2 t^3 + \frac{1}{5}t^5, & t < \beta_i, \\ \frac{8}{15}\beta_i^5, & t \ge \beta_i, \end{cases} \quad (4)$$

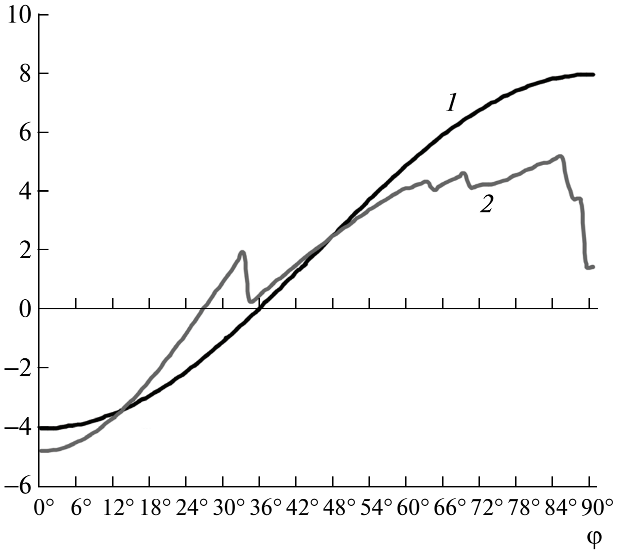

Fig. 2. Latitudinal dependence of dimensionless acceleration of the Earth's inertia moment for a homogeneous Earth (*1*) and heterogeneous Earth (*2*). Latitude is laid off as the horizontal axis.

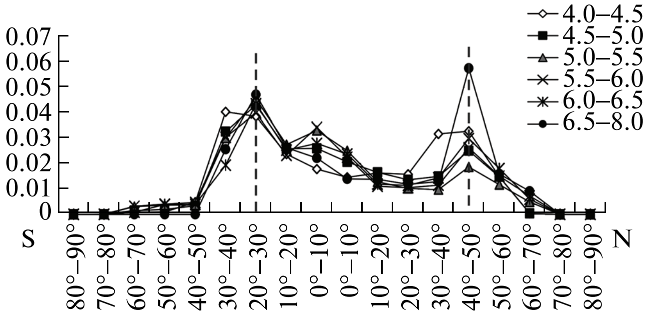

Fig. 3. Latitudinal distributions of the number of seismic events on the Earth for six different magnitude ranges. The size of the latitudinal band is 10. The relative number of the seismic events normalized by the length of the boundaries between lithospheric plates in each seismic band is laid off as the vertical axis. Dashed lines denote two maxima on the latitudinal distributions.

where ρ is the mean density of the Earth and J_0 is determined in (3).

In Eq. (4) the second additive in parenthesis takes into account the inhomogeneity of the internal structure of the Earth by the density and thickness of the layers composing the Earth. Therefore, the critical geocentric latitude would depend on the number of layers of different densities and on the relation between these densities. We note that the value of the geocentric latitude would also depend in this case on the degree of compression of the ellipsoid shaped layers composing the Earth. If $\gamma_i = 1$ for all $i = 1, 2, ..., n$, then equations (2) and (4) coincide.

The authors considered three models of the density distribution for the inhomogeneous Earth: the model of Bullen-Haddon [2, 8], the PREM model [7], and the model AK135-F [9]. The critical geocentric latitudes φ^* calculated for these models appeared equal to 26°18'36", 26°20'16", and 26°17'24", respectively. These values practically coincide.

Figure 2 shows the graphs of dimensionless acceleration for the variations in the Earth's inertia moment for the cases of homogeneous and heterogeneous surface density. The crossings of the abscissa axis with the graphs in this figure correspond to the critical geocentric latitudes. We note that the maximum jump of acceleration from the inertial moment variation on the graph for inhomogeneous Earth corresponds to the boundary between the outer core and lower mantle at a latitude of $\varphi = 33°6'10''$. A sharp density variation in the *dz* layer is characteristic of this place.

The previous calculations of the Earth's inertial moment as a function of the geocentric latitude of the planet demonstrated the existence of the critical latitude where the decreasing variation in the inertial moment changes the acceleration sign from negative to positive.

The analysis of the distribution of earthquake numbers and released energy by latitudes [1, 10] and the distribution of fractures in the Earth's crust [3, 4] demonstrates that increased seismic activity exists precisely within these limits of the geocentric latitudes.

The results of the analysis of latitudinal and depth distributions of earthquakes, which the authors performed on the basis of processing the ISC catalogue (more than 250 000 events with $M \geq 4$ since 1964), are presented in [1]. It was found that the seismic activity of the planet is almost absent at the poles and polar caps of the Earth. It has a clearly pronounced maxima at the mid-latitudes of the northern and southern hemispheres (30°–50°), and a stable local minimum near the equator. The stability of the obtained distribution in time and with the changes in the size of the latitudinal belt is demonstrated. The bimodal form of the distribution described here is characteristic of deep and crustal events. It is characteristic of the number of seismic events and energy released as a result of the earthquakes.

Figure 3 demonstrates the latitudinal distributions of the number of seismic events. It is not possible to explain the results from the point of view of the theory of lithospheric plate tectonics. Tidal forcing and variations in the velocity of the Earth's rotation were possible causes governing the seismicity increase.

Figure 4 presents the graphs of the variation rate (or variations in the latitudinal gradient) of the Earth's inertia moment. These are bimodal functions with maxima at latitudes ±35° (in the case of the homogeneous Earth) and ±27° (for the inhomogeneous Earth). Increased seismic activity exists exactly within the limits of the geocentric latitudes. It follows from

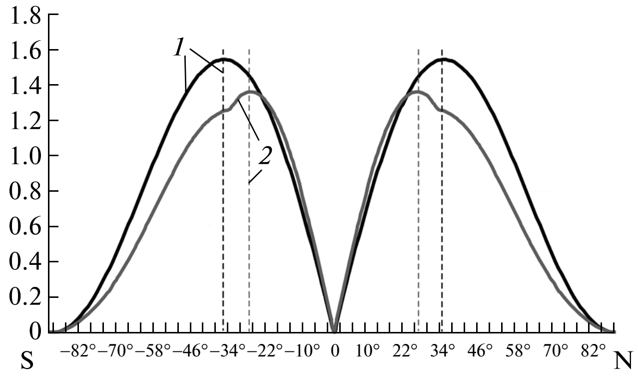

Fig. 4. Latitudinal dependence of the dimensionless velocity of variation (or latitudinal gradient) of the Earth's inertia moment (curve *1* for homogeneous Earth; curve *2* for heterogeneous Earth). The maximum values on the graphs corresponding to the critical latitudes are denoted with dashed lines.

the results of this work that the geometrical form of the figure and density distribution in the Earth's interior can cause the location of hydrodynamic instability and the associated peculiarity in the latitudinal distribution of seismicity on the planet.

It is worth noting that the celestial bodies consisting of gas or fluid matter are characterized by so-called differential rotation when the central part of the body has a greater angular velocity than the polar regions. Such a rotation regime was recorded on the Sun, Jupiter, and Saturn. However, this phenomenon expects a physical explanation.

In the real inhomogeneous Earth [2], the mass of the solid matter (solid inner core, upper mantle, and crust) comprise less than 22%, while the proportion of the fluid and elastic plastic material (fluid core and lower mantle) is approximately 78%. It is worth noting that the long process of the Earth's evolution should have formed a differential regime of the planet's rotation as a celestial body with a complex internal structure. The geological and geophysical observations [6] repeatedly confirmed that a network of fractures of latitudinal orientation, local structures, and other objects exist in the lithosphere that evidence the reality of the existence of the zones of hydrodynamic instability at mid-latitudes.

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