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Residual Hydrodynamic Fields near a Tsunami Source

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Among all tsunami events known at the present time, approximately 80% were caused by a strong underwater earthquake [1]. The decision about announcing a tsunami warning is taken if the magnitude of the seismic event exceeds a specific threshold value. The threshold value can vary slightly depending on the region, but usually it is $M = 7$ [2].

The strong dependence of the tsunami characteristics on the mechanism of the earthquake source and its depth [3] leads to the fact that not every underwater earthquake with magnitude $M \geq 7$ is accompanied by the generation of waves that cause a real danger. An exact calculation of the bottom deformation in the tsunami source is not possible in an operative regime. Therefore, confirmation or cancellation of a tsunami alarm requires objective information about the fact of wave generation. Such information can be obtained when the wave is recorded at the bottom of the coastal sea level station closest to the source.

In recent years, the possibility of applying satellite altimeters for this purpose has been discussed as an alternative to the traditional methods of tsunami recording. However, among the multitude of such attempts, only one tsunami record on December 26, 2004, in the Indian Ocean appeared really successful [4]. The wave amplitude of this rare powerful event reached a significant value (0.8 m) in the open ocean. This made the wave easily recognized against the background of the natural "noise" (~0.1 m) of the variations of water surface location related to mesoscale oceanic eddies. The major part of a dangerous tsunami can appear unrecognizable over the background of the natural "noise."

The existing state of the operative tsunami prediction evidences the fact that any additional information indicating the fact of the generation and characteristics of the tsunami wave should be used. We suggest using residual hydrodynamic fields that appear near the tsunami source as such additional information.

The main mechanism of tsunami generation by an earthquake is related to the displacement of a water volume by the residual bottom deformation. A typical value of the displaced volume during tsunamigenic earthquakes is a few cubic kilometers but sometimes it can exceed 10 km^3 [3, 5]. As the wave propagates from the source region, the displaced volume is distributed (spreads) over the surrounding region. This process is accompanied by notable residual displacements of water particles in the horizontal direction.

In addition, as was shown in [6, 7], a geostrophic eddy should appear over the tsunami source due to the rotation of the Earth. It is noteworthy that the intensity of the eddy motion is unambiguously related to the volume of the displaced water, i.e., to the power of the tsunami source. The direction of rotation indicates the polarity of the bottom deformation. The bottom uplift should be accompanied by the generation of an anticyclonic eddy (clockwise rotation in the Northern Hemisphere), and a subsidence of the bottom is associated with cyclonic rotation.

In the majority of cases, the horizontal size of the tsunami source exceeds significantly the ocean depth. Therefore, both types of residual hydrodynamic fields should develop in the entire ocean column. This should be considered as a property facilitating the recording of these fields in natural conditions. Recording of the fields is possible using drifters with GPS positioning. We specially note that the technical possibility of such measurements became available only in the last few years. In addition, to our knowledge the possibility of tsunami identification based on residual hydrodynamic fields was not considered by anyone for the purposes of operative tsunami forecasting.

In order to avoid confusion, we note that there were attempts using GPS buoys for tsunami recording [8, 9]. However, in the publications that we know, the authors consider recording vertical displacements of the buoys during the propagation of tsunami waves. The method we suggest is based on the recording of the horizontal displacements of water particles, which exceed significantly the vertical displacements of the free surface.

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The residual hydrodynamic fields that appear near the tsunami source exist together with the background currents of different kinds. Therefore, they are not completely residual. Recording of the residual fields (and their reliable distinguishing against the background of other phenomena) is not an easy problem and should be solved beyond the objectives of this work.

This work is dedicated to the development of a method for the calculation of the residual hydrodynamic fields for real tsunami sources and to the application of this method for estimating the residual horizontal displacements of the water particles and velocity fields in a geostrophic eddy on the example of the tsunami on November 15, 2006 (Central Kuril Islands). Such calculation should estimate the values that can be measured in real natural conditions and demonstrate the principal possibility of their recording.

We neglect the curvature of the Earth's surface and consider the problem in the Cartesian rectangular coordinate system. We direct the $0x$ axis to the east and the $0y$-axis to the north. We construct the mathematical model on the basis of the long wave theory, which is widely used in tsunami modeling [1]. The continuity equation and linearized dynamic equations with account for the Coriolis force are written as

$$\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = \frac{\partial \eta}{\partial t} - \frac{\partial \xi}{\partial t}, \tag{1}$$

$$\frac{\partial M}{\partial t} = -gH\frac{\partial \xi}{\partial x} + fN, \tag{2}$$

$$\frac{\partial N}{\partial t} = -gH\frac{\partial \xi}{\partial y} - fM, \tag{3}$$

where ξ is the deviation of the free surface from the equilibrium, $M = uH$ and $N = \nu H$ are components of the total water flux along axes $0x$ and $0y$, u and ν are the corresponding components of the velocity, η is the vertical bottom deformation, H is the ocean depth, g is acceleration due to gravity, $f = 2\omega \sin\alpha$ is the Coriolis parameter, ω is the angular velocity of the Earth's rotation, and α is the latitude.

We assume that the bottom deformation occurs in the time interval $0 < t < \tau$. Before the beginning of the bottom deformation at time moment $t = 0$, the water layer was at rest:

$$M = N = \xi = \eta = 0. \tag{4}$$

According to the Helmholtz theorem, an arbitrary vector field can be presented as a superposition of the potential and eddy components, i.e., as the sum of the gradient of the scalar potential and curl of the vector potential. The components of the two-dimensional vector field of the total flux can be expressed by means of the potential φ and stream function ψ in the following form:

$$M = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad N = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}. \tag{5}$$

We substitute presentation (5) into continuity equation (1) and get the equation from which we automatically exclude the stream function

$$\Delta\varphi = \frac{\partial \eta}{\partial t} - \frac{\partial \xi}{\partial t}. \tag{6}$$

Let T be the time moment when the tsunami waves leave the domain considered here ($T > \tau$) and the sought residual fields are established. We integrate equation (6) with respect to time from 0 to T with account for the initial conditions (4) and obtain the Poisson equation

$$\Delta\Phi = \eta_T - \xi_T, \quad \Phi \equiv \int_0^T \varphi dt, \tag{7}$$

where η_T is the residual bottom deformation and ξ_T is the residual displacement of the water surface over the region of the bottom deformation corresponding to the geostrophic eddy. According to the estimates obtained in [7], in the real conditions we get $|\xi_T| \ll |\eta_T|$. Therefore, it is reasonable to neglect ξ_T in the right part of equation (7) and use the following equation for practical calculations

$$\Delta\Phi = \eta_T, \tag{8}$$

which allows us to determine the horizontal residual displacements of water particles caused by the potential part of the current from the known residual bottom deformation $\eta_T(x, y)$ and distribution of depths $H(x, y)$

$$X_T = \frac{1}{H}\frac{\partial \Phi}{\partial x}, \quad Y_T = \frac{1}{H}\frac{\partial \Phi}{\partial y}. \tag{9}$$

If the tsunami source is located at a significant distance from the coast, equation (8) can be considered on an unlimited plane $0xy$. In this case, the solution of the Poisson equation is given by the known analytical relation. Then, a very rare situation is realized when the problem for the basin with real (arbitrary) bathymetry can be completely solved analytically. However, in the majority of cases, the tsunami sources are located quite close to the coast and it is not correct to neglect the influence of the costal line. Then, the problem should be solved numerically and the boundary condition should be specified at the coastline (or fixed isobath), which is a consequence of the zero flow condition:

$$\frac{\partial \Phi}{\partial \mathbf{n}} = 0, \tag{10}$$

where $\mathbf{n}$ is the normal to the coastline.

It is worth noting that we do not limit ourselves to consideration of small wave amplitudes in the determination of the residual horizontal displacements of water particles. We neglect only the residual displacement of the water surface in the geostrophic eddy and assume that the water edge line after the interaction between the tsunami and the coast returns to its initial position. Both assumptions are quite well justified in practice.

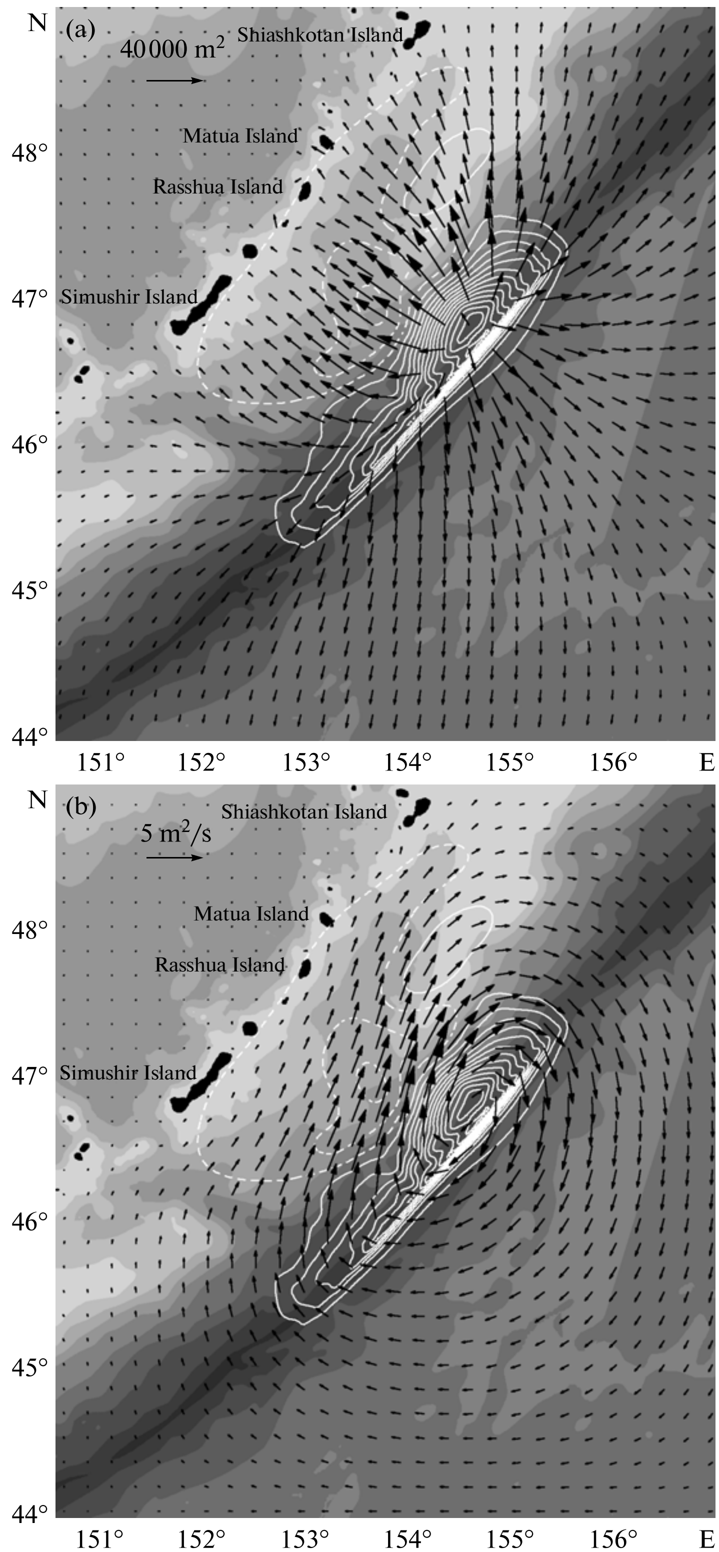

Fig. 1. Potential (a) and eddy (b) components of the residual hydrodynamic field near the tsunami source on November 15, 2006. White contour lines show the vertical deformation of the bottom (the solid line indicates elevation, the dashed line is related to depression, and the interval is 0.2 m). The isobaths are plotted with an interval of 1000 m. The vector fields are shown with black arrows whose length is proportional to the horizontal displacement of the water particles (a) and the velocity of the eddy current (b), which are multiplied by the ocean depth at the given point.

Then, we obtain the equation for the calculation of the stream function. We shall assume for simplicity that the Coriolis parameter f does not depend on latitude and it is a constant within the calculation domain. We exclude the perturbation of the free surface ξ from equations (2) and (3) using the standard procedure (cross differentiation with respect to coordinates x and y). We make a transition in this expression from the total fluxes to the stream function and potential. Then we integrate it with respect to time from 0 to T and obtain

$$H\Delta\psi_T - fH\Delta\Phi - \frac{\partial\psi_T}{\partial y}\frac{\partial H}{\partial y} - \frac{\partial\psi_T}{\partial x}\frac{\partial H}{\partial x}$$
$$+f\left(\frac{\partial\Phi}{\partial y}\frac{\partial H}{\partial y} + \frac{\partial\Phi}{\partial x}\frac{\partial H}{\partial x}\right) + f\int_0^T\left(\frac{\partial\psi}{\partial y}\frac{\partial H}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial H}{\partial y}\right)dt = 0. \quad (11)$$

We note that, in the case $H = \text{const}$, equation (11) is simplified significantly and can be written as

$$\Delta\psi_T - f\Delta\Phi = 0. \quad (12)$$

It follows from equation (12) that in the case of a flat horizontal bottom the eddy component of the current is actually the residual field, which is constant in time. In the general case, when the ocean depth is a function of the horizontal coordinates, the eddy field would be time dependent. The last term in equation (11) indicates this. However, if the eddy current is adjusted to the forms of the bottom topography (the velocity is directed along the isobaths), the integrand turns to zero. Due to the fact that the value of fT is small, we can neglect the last term in equation (11) for the initial estimate of the eddy field. Then, the equation for the stream function becomes stationary:

$$\Delta\psi_T - \frac{1}{H}\frac{\partial\psi_T}{\partial x}\frac{\partial H}{\partial x} - \frac{1}{H}\frac{\partial\psi_T}{\partial y}\frac{\partial H}{\partial y}$$
$$= f\left(\Delta\Phi - \frac{1}{H}\frac{\partial\Phi}{\partial x}\frac{\partial H}{\partial x} - \frac{1}{H}\frac{\partial\Phi}{\partial y}\frac{\partial H}{\partial y}\right). \quad (13)$$

The boundary condition for the streamline at the coast (zero flux condition) is expressed by the following relation:

$$\psi_T = 0. \quad (14)$$

The velocity components of the eddy current are calculated from the known stream function as follows:

$$u_T = \frac{1}{H}\frac{\partial\psi_T}{\partial y}, \quad \nu_T = -\frac{1}{H}\frac{\partial\psi_T}{\partial x}. \quad (15)$$

We selected one of the strongest events of recent time, the tsunami generated by the earthquake with magnitude $M_W = 8.3$ on November 15, 2006, as an example for the calculation of the residual hydrodynamic fields. The expedition investigation in the summer of 2007 on the coast closest to the source revealed runup heights up to 20 m [10].

Equations (8) and (13) were solved using the finite-difference implicit (iteration) method. The residual vertical bottom deformation was calculated according to the data about the structure of the bottom motion, which are available in digital format on the server of the US Geological Service [5]. The bathymetry of the calculation domain was prepared on the basis of the GEBCO 1-minute digital atlas (British Oceanographic Data Centre) and digital nonclassified marine navigation charts of the Main Administration of the Navigation and Oceanography of the Ministry of Defense of the Russian Federation (GUNiO).

Figure 1 presents the results of numerical calculation: the structure of the residual hydrodynamic fields near the tsunami source. Due to the strong variations in the ocean depth in the calculation domain, the amplitudes of the displacement vectors (X_T, Y_T) and velocity vectors of the eddy current (u_T, ν_T) vary in the range of a few orders of magnitude. Therefore, the potential and eddy fields are shown in terms of the total fluxes. Figure 1a demonstrates the vector field $(X_T H, Y_T H)$, and Fig. 1b demonstrates the vector field $(u_T H, \nu_T H)$.

It is seen from Fig. 1a that the amplitude of vector $(X_T H, Y_T H)$ reaches 4×10^4 m^2. In the region of the deep ocean (a few thousand meters), this corresponds to a horizontal displacement of a few meters. In the shallow ocean, the residual horizontal displacements can exceed 100 m. It is clear that such significant displacements can be easily recorded by the drifters with GPS positioning. A preliminary calculation of the fields of the residual displacements from the set of hypothetical tsunami sources can facilitate the optimum choice of the region for the deployment of drifters.

Analysis of the eddy field plotted in Fig. 1b allows us to state that the amplitude values of the vector of the total flux do not exceed 5 m^2/s. In the deeper ocean, this value recalculated to the current velocity corresponds to a velocity of $\sim 10^{-3}$ m/s. In shallow places, the velocity can increase up to 10^{-2} m/s. The estimate of the kinetic energy of the eddy current gives a value of 6.4×10^{10} J, which is 0.066% of the potential energy of the initial elevation in the tsunami source (9.7×10^{13} J [5]). On the one hand, such small velocities of the eddy current cause difficulties in the use of the geostrophic eddies for the operative tsunami forecast. On the other hand, the justified possibility of neglecting the displacement of the drifters by the eddy current simplifies the interpretation of the data on the displacements of drifters related to the potential field.

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